LIGHT-NEUTRINO MASS HIERARCHIES, AND THE NEUTRINOLESS ββ DECAY

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Abstract

Experimental data for neutrino flavor oscillations and neutrino mixing mechanisms, are used to calculate the effective neutrino mass relevant for the neutrinoless double-beta decay. The observability of the neutrinoless double beta decay mode, in the decay of 76Ge is discussed, with reference to reported claims about positive signals.
Key words: neutrino physics, nuclear double beta decay

Resumen

Neutrinos livianos y su rol en el decaimiento beta doble. Datos experimentales recientes, referidos a oscilaciones entre neutrinos de diferente sabor, son utilizados para calcular la masa del neutrino (sabor electrón), que es la cantidad relevante en el decaimiento beta doble. A partir de los resultados obtenidos se discute la posibilidad de observar dicho decaimiento.
Palabras clave: física del neutrino, decaimiento beta doble en núcleos.

1. Introduction

The knowledge about the properties of the neutrino has been dramatically advanced by the results of measurements performed by various large-scale experiments [1, 2, 3, 4, 5]. These experimental evidences have confirmed the existence of neutrino flavor oscillations and have set stringent limits to the neutrino mass-mixing mechanism [6, 7, 8]. Detailed discussions about the extracted values of the mixing angles, mixing amplitudes, and mass differences have been presented in [9, 10, 11]. The implications of the latest results on the physics of electroweak interactions and dark-matter studies have been discussed in [12]-[14].

In addition to the findings on neutrino flavor oscillations and the confirmation of some of the theoretically predicted possibilities for the mixing and enhancement of the oscillations in the presence of matter [15], double-beta-decay experiments can provide complementary information on the nature of the neutrino and about its absolute mass scale [16]-[20]. This is a unique feature of the double-beta decay, which must be consistent with other scalefixing measurements [5]. In the case of double-beta-decay measurements the knowledge about relevant nuclear matrix elements is crucial, as it is crucial to know the correct neutrino-mass spectrum for the analysis of the other type of measurements. The implications of the results of the solar, atmospheric, reactor and astrophysical neutrino experiments upon double-beta-decay experiments have been stated already in several publications, see for instance [21]-[26]. To the wealth of parameters involved in the analysis, like CP-phases, mixing angles and masses, one should add the nuclear-structure degree of freedom needed to extract the effective electron neutrino mass [17]-[20].

At first glance, to physicists who are less familiar with nuclear-structure analysis, it may appear simple to produce the needed nuclear-structure information. Unfortunately it is not so because of several reasons:

a) Double-beta-decay transitions take place in medium –and heavy– mass systems, where explicit shell-model calculations are unfeasible, unless severely truncated valence spaces are used.

b) The sensitive part of the calculations depends on the information about the structure of the states of the double-odd-mass nuclei. These intermediate states play an essential role in the second-order transition matrix elements entering the expression of the decay rate, and less is known about them, as compared with the relatively large amount of information gathered about double-even-mass nuclei.

c) In dealing with medium –and heavy– mass nuclei one has to introduce approximations to obtain the participant wave functions and these approximations are not unique, they vary from model to model.

d) To assign a certain degree of significance to the already existing theoretical results one has to define, first, what should be taken as the equivalent of the experimental confidence level, e.g. which models may be taken as references and what would be the confidence level assigned to them depending upon the used approximations.

In the past, all of these features have been referred to as the *uncertainties* in the nuclear matrix elements and roughly estimated to be within factors of 2 to 3, with respect to some reference values. This aspect of the problem certainly deserves some attention, as we are going to discuss later on in this work, since there turns out to be a gap between the range of masses extracted from double-beta-decay studies, 0.4 eV to 1.7 eV [22], and those extracted from the other neutrino-related studies which yield upper limits of the order of 0.10 to 0.20 eV [24] or even lower [12]. There is a clear discrepancy between both sets of results concerning the observability of neutrinoless double-beta decay. This issue has become a hot one, due to the recent claim [27] about the positive identification of neutrinoless-double-beta decay signals in the decay of 76Ge (see also [28, 29]), from which a central value of the mass of the order of 0.39 eV was extracted [27]. We think that these aspects must be considered from points of view of both the neutrino physics and the nuclear-structure physics. To achieve this goal, in this work we discuss the constraints set by the oscillation and mass parameters on the effective electron neutrino mass and compare these neutrino masses with the ones coming from the analysis by using nuclear-structure information. We start from the best-fit mass-mixing matrix presented in [30] and from other estimates of the mixing matrix, i.e. the parametrization in terms of the mixing angle of solar neutrinos, and the estimation based on a maximum-mixing scheme [23].

In the first part of the paper we review the basic elements of the theory and discuss the structure of the adopted neutrino mass-mixing matrix. We discuss a way to extract light-neutrino masses from the observed mass differences and by combining them with the adopted neutrino mass-mixing matrix we calculate the effective neutrino mass relevant for the neutrinoless double beta decay. In the second part of the paper we review the current nuclear structure information about the neutrinoless double beta decay, by presenting the up-to-date values of the effective neutrino mass extracted from the adopted limits on the half-lives. In doing so, we have considered the range of variation for the nuclear matrix elements, calculated within definite classes of models. The analysis covers the calculated values of the nuclear matrix elements during the last fifteen years. This information is needed to estimate the plausibility of future double-beta-decay experiments. Finally, we discuss the observability of the neutrinoless double beta decay in the context of the present results. Details of the calculations are presented in [59].

2. Neutrino oscillation data

Two- and three-generation analysis of neutrino data, provided by the solar and atmospheric observations and by the range of mass differences explored in reactor-based experiments, have been performed by several groups [6]-[11]. The picture which emerges from these very detailed analyses of neutrino-flavor oscillations favours the Large Mixing Angle (LMA) solution of the Mikheyev-Smirnov-Wolsfenstein (MSW) mechanism [15]. Recently, the KamLAND collaboration [3] has confirmed the LMA solution and a crucial step towards the elucidation of the neutrino mass hierarchies was given by the results of WMAP [5, 24], which fix a stringent upper limit for the scale of neutrino masses. A brief compilation of the adopted results is given in Table 1. As shown in this table, the SNO data are consistent with a value of the mass difference of the order of 10^{-5} eV^2 (solar-neutrino data), and another independent scale, of the order of 10^{-3} eV^2, has been determined from the analysis of the atmospheric-neutrino data, which is in the range of the sensitivity of the reactor-based measure-

ments. Because of the independence of the determined mass differences, the global picture is consistent with the existence of three active neutrino flavors. To these data, the information obtained by WMAP is adding the value of the upper limit of the sum of the three mass eigenvalues (light-neutrino masses only), which is of the order of 0.71 eV [5].

To calculate effective neutrino properties, like the effective electron-neutrino mass, one needs to know the neutrino-mixing matrix and the light-neutrino mass hierarchy [16]. The determination of the elements of the mixing matrix, and the absolute values of the masses is the ultimate goal of any of the models of the neutrino and it is, of course, a matter of intensive effort. Out of the very rich, recently published list of articles dealing with the analysis of the SNO results, we have selected the results presented in the paper of Bandyopadhyay, Choubey, Goswami and Kar (BCGK) [30], together with the expression of the mixing matrix in terms of the solar-neutrino data, and the zeroth-order approxima-

tion of the mixing matrix assuming maximum mixing, to perform our calculations. Our choice is motivated by the fact that in the BCGK paper the best-fit value of the mixing matrix, with respect to the solar, atmospheric, and CHOOZ data, is written explicitly and the confidence level of the results is well established. By performing a three- generation statistical analysis of the solar-neutrino and CHOOZ data, and by considering the current values of the solar and atmospheric neutrino mass differences, the authors of BCGK found that the best fit occurs in the LMA region. This finding greatly simplifies the form of the neutrino mixing matrix, because it narrows the value of of the mixing with the 3-mass eigenvalue down to a very small range around 0.

The next step consists of the definition of a neutrino mass hierarchy, that is the relative order of the mass eigenvalues, which cannot be fixed only by the measured squared mass differences. In Table 2 we give, for each of the adopted forms of the neutrino mixing matrix, the range of values of the calculated effective electron neutrino masses. As it can be seen from this table, the largest value which one can obtain for the electron neutrino mass is of the order of 0.24 eV, and the smallest one is of the order of 10^{-5} eV, both for the degenerate mass hierarchy. Notice that the larger value is of the order of the mass scale extracted from the results of WMAP and it will certainly depend upon new results for the neutrino contribution to the Universe (visible) mass density. A value of this density smaller than 0.5 eV would then give a mass of the order of 0.16 eV, while the estimate 0.18 eV [12] will reduce it to the more stringent value of 0.06 eV.

Table 1. Current limits on neutrino-mass differences. The values listed are a compilation of the results from the SNO [1], SK [2], KamLAND [3] and WMAP [5].

$\delta m^2_{12} = \delta m^2_{solar}$	$5 \times 10^{-5} eV^2 \rightarrow 1.1 \times 10^{-4} eV^2$
$\delta m^2_{23} = \delta m^2_{atm}$	$10^{-3} eV^2 \rightarrow 5 \times 10^{-3} eV^2$
$\sin^2 2\theta_{solar}$	≈ 0.86
$\sin^2 2\theta_{atm}$	≈ 1.0
Ω_v	< 0.71 eV

Table 2. Calculated effective electron-neutrino masses $<m_v> \pm$. Indicated in the table are the mass hierarchy and the adopted mixing matrix. The values are given in units of eV. The adopted values for the mass differences are $\delta m^2_{12} = 7.1 \times 10^{-5} eV^2$, $\delta m^2_{23} = 2.7 \times 10^{-3} eV^2$, and $m_0 = 0.24$ eV. The mixing matrix U(a) is taken from the best fit of [30], U(b) is based on the largest values of the solar and atmospheric mixing angles, and U(c) assumes maximum mixing

Hierarchy		$< m_v >$	U(a)	U(b)	U(c)
Normal	$(m_1 = 0)$	$< m_v >-$	-0.010	-0.012	-0.019
		$< m_v >+$	0.011	0.012	0.019
	(extreme)	$< m_v >-$	0.105	0.086	-0.769×10^{-4}
		$< m_v >+$	0.231	0.231	0.231
Inverse	$(m_3 = 0)$	$< m_v >-$	0.105	0.087	-0.153×10^{-2}
		$< m_v >+$	0.234	0.235	0.235
	(extreme)	$< m_v >-$	0.108	0.088	-0.749×10^{-4}
		$< m_v >+$	0.237	0.237	0.237
Degenerate	(extreme)	$< m_v >-$	0.107	0.088	-0.715×10^{-4}
		$< m_v >+$	0.237	0.237	0.237

3. Nuclear matrix elements

The implication of these results for the electron neutrino mass upon the rates of the neutrinoless double beta decay, is easily seen if one writes[1] the corresponding half-life[17], in terms of nuclear matrix elements and phase-space factors, both entering the mass term of the transition probability [17].

There are several aspects concerning the analysis of neutrinoless double beta decay transitions which should be mentioned,

a) In the event of a successful measurement of neutrinoless double beta decay and considering the improving information emerging from neutrino-related measurements, the calculation of the half-life may be viewed as a crucial test for nuclear models,

b) If one assigns a certain confidence level to nuclear-structure calculations, by fixing the value of the nuclear matrix elements, and takes the range of values of the effective neutrino mass extracted from neutrino-related measurements, calculated half life may be viewed as a criterium for determining the observability of neutrinoless double beta decay.

c) In the event of a positive measurement of neutrinoless double beta decay, and considering a reliable estimate of the nuclear matrix elements, the half life may be viewed as a consistency equation for the effective neutrino mass seen in double-beta-decay as compared with the one extracted from neutrino-related experiments.

Let us start with the discussion of the nuclear-structure related information, contained in the nuclear matrix elements. The ultimate goal of nuclear-structure models is, in fact, the prediction of observables based on the knowledge about nuclear wave functions down to the needed level of accuracy. In the case of $0\nu\beta\beta$ decay studies, to achieve this ultimate goal one needs to surpass several requirements, some of which are purely technical and some of which are conceptual. Among the technical barriers one has, of course, the un-feasibility of large-scale shell-model calculations, prohibited by hardware constraints. Among the conceptual requirements one has the realization that a prediction of a neutrinoless double-beta-decay rate should always be accompanied by other model predictions, like single-beta-decay, electromagnetic and particle-transfer transitions involving the nuclei which participate

in the double-beta-decay transition under consideration. We stress the point that, in our experience, the study should be conducted on the basis of a case-by-case analysis.

Most of the current nuclear-structure approximations are based on the proton-neutron quasiparticle random phase approximation (pnQRPA). This is a framework where proton-neutron correlations are treated as basic building blocks to describe the nuclear states which participate in a double-beta decay transition. The pnQRPA formalism is rather well known and it has been discussed in a large number of publications during the last forty years. For the sake of brevity we are not going to present it here again, rather we would like to refer the interested reader to [17] for details. In particular, the sensitivity of the pnQRPA method to values of specific parameters of the interactions, like the sensitivity to the renormalization of the particle-particle (proton-neutron) coupling, has been a matter of intensive studies. Again, we would like to refer to [17] for details concerning this point as well as concerning the large number of extensions of the pnQRPA method, their successes and failures. Restricting ourselves to a very elementary theoretical background, we can say that the standard procedure, applied in the literature to calculate the neutrinoless double-beta decay rate, involves three major components:

a) The calculation of the spectrum of the intermediate double-odd-mass nucleus with (A, N≠1, Z±1) nucleons. The pnQRPA is an approximate diagonalization in the one-particle-one-hole, 1p-1h, (or two-quasiparticle) space and it includes the effects of 2p-2h ground-state correlations by means of the backward-going amplitudes. Since the calculations are based on a quasi-particle mean field one forces the breaking of certain symmetries, like the particle-number symmetry by the use of the BCS approximation, and the isospin symmetry, by the use of effective proton and neutron single-particle states. The final results of the pnQRPA calculations will certainly be affected by these symmetry-breaking effects induced by the way in which we handle the nuclear interactions. Some attempts to cure for these effects have been implemented by means of enlarging the representation space, including Pauli-principle-related blockings and by performing self-consistent approaches beyond the quasiparticle mean field. As said before, the list of various extensions of the standard pnQRPA is too long to be commented here in detail. A fairly complete list of references about the set of extensions of the pnQRPA is given in [17]-[19]. We will generally refer to these approximations as pnQRPA-

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[1]Only the mass sector of the half-life will be considered in the following analysis. Complete expression of the half-life, including right-handed currents, can be found in [17].

related ones. In this paper we shall show the results based on this family of approximations[2]. In addition, we quote the results of the available shell-model calculations.

b) The calculation of the leptonic phase-space factors, as dictated by the second-order perturbative treatment of the electroweak interaction. At the level of the minimal extension of the Standard Model (SM) lagrangian (mass sector only), these phase-space factors can be easily calculated, and the values of them should be rather universal causing no source of discrepancies in the calculations, except for the adopted value of the axial-vector coupling. At the level of the two-nucleon mechanism this value is currently fixed at $g_A=1.254$ but for the medium-heavy and heavy nuclei an effective value of $g_A=1.0$ has also been used. In this work we adopt the conservative estimate of $g_A=1.254$. Expressions for the phase-space factors, for theories beyond the minimal extension of the SM lagrangian, i.e. for left-right and right-right couplings, have been listed exhaustively in the literature (see e.g. [17]-[18]) and their values are well defined, too. In going beyond the two-nucleon mechanism one has to consider, also, the momentum dependence of the operators, which will reflect upon the structure of the phase-space factors. This is also true for the case of calculations where one is including p-electron wave effects and/or forbidden decays.

c) The calculation of the matrix elements of the relevant current operators which act upon the nucleons. These operators are also well known and their multipole structures are derived from the expansion of the electroweak current [17]. In the present calculation we have considered the standard type of operators, without introducing further momentum dependences in them, as originating from the electroweak decay at the quark level [31].

A compilation of the values of nuclear matrix elements and phase-space factors can be found in [17]. The current information about the status of $0\nu\beta\beta$ decays is reported in [32]-[36].

Tables 3 and 4 show the set of double-beta-decay systems where experimental searches for signals of the $0\nu\beta\beta$ are conducted at present or planned for the next generation of double-beta-

decay experiments. The tables contain the experimental lower limits for the neutrinoless double beta decay half-life [37]-[46], the full range of variation of the nuclear matrix elements, predicted by different models [17], the calculated phase-space factors, and the extracted values of the effective neutrino mass.

In Table 4 only a sub-group of calculations are presented, namely the ones based on the plain spherical pnQRPA approach of [17] (third column). These results are compared with our present calculations shown in the fourth column.

In the following, some brief details about the present pnQRPA calculations are given. They have been done by following the procedure outlined in [17]. The two-body nuclear interactions were constructed by using the G- matrix interaction of the Bonn type including two to three major harmonic- oscillator shells around the proton and neutron Fermi surfaces. The spherical Woods-Saxon potential was used to generate the single-particle energies and small adjustments of these energies were done in the vicinity of the Fermi surfaces to reproduce the low-energy quasiparticle spectra of the adjacent odd-mass nuclei. Following the criteria which we have advanced above, the various parameters involved in the calculations have been fixed by reproducing the known data on single-beta-decay transitions around the nuclei of interest for the double-beta-decay transitions which we are considering here. No further adjustment of the proton-neutron particle-particle coupling constant [17] is introduced once the known single-beta decay observables are reasonably reproduced.

As one can see, our present results are in good agreement with the other pnQRPA calculations, except for 136Xe where our calculation gives a larger matrix element than the other calculations. This deviation might occur due to the semi-magic nature (the neutron shell is closed) of 136Xe, forcing the transition from the two-quasiparticle description to the particle-hole description.

If one compares the extracted neutrino masses of Table 3 with the ones given in the previous section, it becomes evident that the present generation of neutrinoless double beta decay experiments is rather insensitive to the effective neutrino mass coming from the best fit of the solar+atmospheric+reactor data, except for the Heidelberg-Moscow experiment if one takes the range of values between 0.11 eV and 0.56 eV reported in [27]. If one takes the largest possible effective mass, 0.24 eV, which is favored by the inverse and degenerate mass hierarchies (see Table 2), one sees that it is outside the range of the present limits fixed by double-beta-decay

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[2] We shall explicitly quote the sources from where the results have been taken in order to avoid here a repetition of the details of each formalism, since the aim of the present section is not to present a critical review of the theories but rather to show their results to give an idea about the spread in the values of the relevant nuclear matrix elements.

Table 3. $0\nu\beta\beta$: Model-dependent estimates and experimental limits. The double-beta-decay systems are given in the first column. The factors $C_{mm}^{(0v)}$ are given in units of yr^{-1} and their values are shown within the intervals predicted by different nuclear-structure models, like the Shell Model (a), the Quasiparticle Random Phase Approximation (b), the pseudo SU(3) model (c), and various other models (d). The value $g_A = 1.254$ is used. The quantities $t_{1/2}^{(0v)}$ are the experimental lower limits of the half-lives, in units of years. The corresponding references are quoted in brackets. The factor F_N (lower limit) is shown in the fourth column and the values are given within the intervals provided by the factors $C_{mm}^{(0v)}$. The last column shows the range of variation of the extracted effective neutrino mass (upper limits) in units of eV. The coefficients $C_{mm}^{(0v)}$ are taken from [17], except for the case of 124Sn [58].

System	$C_{mm}^{(0v)}$			$t_{1/2}^{(0v)}$			F_N		$<m_v>$
^{48}Ca	(1.55-4.91)	10^{-14}	(a)	9.5	10^{21}	[37]	(1.47-4.66)	10^8	(23.7-42.1)
	(9.35-363)	10^{-15}	(b)				(8.88-345)	10^7	(8.70-54.2)
^{76}Ge	(1.42-28.8)	10^{-14}	(d)	2.5	10^{25}	[29]	(3.55-72.0)	10^{11}	(0.19-0.86)
^{82}Se	(9.38-43.3)	10^{-14}	(d)	2.7	10^{22}	[39]	(2.53-11.7)	10^9	(4.73-10.2)
^{96}Zr	(9.48-428)	10^{-15}	(b)	1.0	10^{21}	[40]	(9.48-428)	10^6	(24.7-166)
^{100}Mo	(0.07-2490)	10^{-15}	(b)	5.5	10^{22}	[41]	(0.38-13700)	10^7	(1.38-262)
^{116}Cd	(5.57-66.1)	10^{-14}	(b)	7.0	10^{22}	[42]	(3.90-46.3)	10^9	(2.37-8.18)
^{124}Sn	(2.29-5.70)	10^{-13}	(b)	2.4	10^{17}	[43]	(5.50-13.7)	10^4	(1.38-2.18) 10^3
^{128}Te	(1.71-33.6)	10^{-15}	(b)	8.6	10^{22}	[44]	(1.47-28.9)	10^8	(9.51-42.1)
^{130}Te	(1.24-5.34)	10^{-13}	(b)	1.4	10^{23}	[44]	(1.74-7.48)	10^{10}	(1.87-3.87)
^{136}Xe	(2.48-15.7)	10^{-14}	(a,b)	4.4	10^{23}	[45]	(1.09-6.91)	10^{10}	(1.94-4.89)
^{150}Nd	(4.78-77.4)	10^{-13}	(b,c)	1.7	10^{21}	[46]	(8.13-132)	10^8	(4.45-17.9)

Table 4. Calculated phase-space factors $G_1^{(0v)}$ and calculated nuclear matrix elements, using the formalism of the spherical pnQRPA, for some of the double-beta emitters included in Table 3. The phase space factors are given in units of yr^{-1} and the dimensionless matrix elements are scaled by the nuclear radius [17]. The third column, indicated as N.M.E., gives the extreme values of the nuclear matrix elements $M_{GT}^{0v} (1 - X_F)$ reported in the literature (see the captions to Table 3), and the fourth column, indicated as N.M.E. (this work), gives the results of the present calculations for $M_{GT}^{0v} (1 - X_F)$. The last column shows the range of values for the effective neutrino mass, in units of eV, extracted from the results given in the third and fourth columns

System	$G_1^{(0v)} \times 10^{14}$	N.M.E.	N.M.E.(this work)	$<m_v>$
^{48}Ca	6.43	1.08-2.38		8.70-19.0
^{76}Ge	0.63	2.98-4.33	3.33	0.30-0.43
^{82}Se	2.73	2.53-3.98	3.44	4.73-7.44
^{96}Zr	5.70	2.74	3.55	19.1-24.7
^{100}Mo	11.30	0.77-4.67	2.97	1.38-8.42
^{116}Cd	4.68	1.09-3.46	3.75	2.37-8.18
^{128}Te	0.16	2.51-4.58		9.51-17.4
^{130}Te	4.14	2.10-3.59	3.49	1.87-3.20
^{136}Xe	4.37	1.61-1.90	4.64	0.79-2.29

experiments, with the possible exception of the decay of 76Ge, which just barely reaches this estimate. Naturally, the values of the electron neutrino mass extracted from the experimental lower limits of the half-life are model dependent, since the connection between the half-lives and the effective neutrino mass is given by the nuclear-model-dependent factors (see Table 3).

Table 5. Calculated nuclear matrix elements for the case of ^{76}Ge. The values $C_{mm}^{(0)}$ are given in units of yr^{-1}. The adopted value for the half-life is the value recommended in [29], $t_{1/2}^{(0_\nu)} = 2.5 \times 10^{25}$yr. Indicated in the table are the models used to calculate the nuclear matrix elements, which are taken from the references quoted in the last row of the table. The abreviations stand for the proton-neutron quasiparticle random-phase approximation (pnQRPA), particle-number-projected pnQRPA (pnQRPA (proj.)), proton-neutron pairing pnQRPA (pnQRPA+pn pairing), the renormalized pnQRPA (RQRPA), the second pnQRPA (SQRPA), the self-consistent renormalized pnQRPA (SCRQRPA), the fully renormalized pnQRPA (full-RQRPA), and the variation after projection mean-field approach (VAMPIR). The model assumptions underlying these theories are presented in the quoted references.

$C_{mm}^{(0)}$	$F_N \times 10^{-12}$	Theory	Reference
1.12×10-13	2.80	pnQRPA	[47, 48]
6.97×10-14	1.74	pnQRPA	[31]
7.51×10-14	1.88	pnQRPA (proj.)	[31]
7.33×10-14	1.83	pnQRPA	[49]
1.42×10-14	0.35	pnQRPA+ pn pairing	[49]
1.18×10-13	2.95	pnQRPA	[50]
8.27×10-14	2.07	pnQRPA	[51]
2.11×10-13	5.27	RQRPA	[52]
6.19×10-14	1.55	RQRPA+ q-dep. operators	[52]
$1.8 - 2.2 \times 10$-14	0.45-0.55	pnQRPA	[53]
$5.5 - 6.3 \times 10$-14	1.37-1.57	RQRPA	[53]
$2.7 - 3.2 \times 10$-15	0.07-0.08	SCRQRPA	[53]
1.85×10-14	0.46	pnQRPA	[54]
1.21×10-14	0.30	RQRPA	[54]
3.63×10-14	0.91	full-RQRPA	[54]
6.50×10-14	1.62	SQRPA	[54]
2.88×10-13	7.20	VAMPIR	[55]
1.58×10-13	3.95	Shell Model	[56]
1.90×10-14	0.47	Shell Model	[57]

With reference to the results shown in Table 4, the span in the effective neutrino mass is smaller (see the last column of Table 4) than when all the available model calculations are included (see the last column of Table 3).

This means that to reach the neutrino-mass value resulting from the neutrino data, one definitely needs larger matrix elements than the ones produced thus far by the spherical pnQRPA model, and/or longer half-lifes than the present measured limits. These observations will be discussed more in the next section.

3.1 *pnQRPA Matrix elements for ^{76}Ge*

Table 5 shows the results of the matrix elements, corresponding to the mass sector of the neutrinoless double-beta decay in 76Ge, calculated within the family of the pnQRPA-related models [47]-[54][3] and effective neutrino masses which were shown, previously, in Tables 3 and 4. The results of the other, pnQRPA-related, approximations seem to be less constant and they deviate more from the central range value. In analyzing the results of [54] one can notice that the largest value does not differ much from the standard pnQRPA value, although is has been obtained by using a more involved approximation. By using the phase-space factors listed in Table 4, we arrive at the central value 3.65, for the matrix elements in the pnQRPA. The corresponding value for the latest large-scale shell-model calculation is 1.74. Therefore, the latest shell-

[3]Notice that the results of [47], which are relevant for the analysis performed in [27], are only 1.3 times larger than the average pnQRPA matrix element.

model results [57] and the centroid of the pnQRPA results differ by a factor of the order of 2. In terms of the effective neutrino mass, using the latest half-life limit recommended in [29], these matrix elements lead to electron neutrino mass values of 0.35 eV and 0.74 eV, respectively.

It means that to go to masses of the order of 0.24 eV, as required by WMAP, one needs larger nuclear matrix elements than the ones given by the pnQRPA or by the available shell-model results. In fact, to reach the WMAP limit one would need the value of the order of 5.4 for the relevant nuclear matrix element, which is approximately 1.4 times larger than the reference pnQRPA value. The largest matrix element listed in Table 5, coming from the VAMPIR approach [55], would yield to a mass of the order of 0.19 eV, a result which just touches the value 0.24 eV coming from the analysis of the neutrino-related data. However, it is appropriate to point out here that the VAMPIR matrix element is to be considered unrealistically large because in the calculations of [55] no proton-neutron residual interaction was included.

Finally, our present value 3.33 (see Table 4) yields an effective neutrino mass of the order of 0.39 eV, if one takes for the half-life the lower limit recommended in [29], and 0.50 eV if one takes for the half-life the value given by the Heidelberg-Moscow collaboration [27].

4. Observability of the neutrinoless double beta decay

To grasp an idea about the observability of the neutrinoless double beta decay in other systems, we can compare the values of Table 3, with the ones obtained by using an effective neutrino mass of 0.24 eV, corresponding. For the case of 76Ge the upper value is given by the unrealistically large nuclear matrix element of [55].

The results of Table 3 indicate a departure with respect to the experimental limits by orders of magnitude, excluding the case of ^{76}Ge which is closer but still outside of the range consistent with the solar+atmospheric+reator neutrino data.

Thus the issue about the observability of the neutrinoless double beta decay relies upon the estimates for the effective neutrino mass and upon the estimates of the relevant nuclear matrix elements. While in some cases the differences between the calculated matrix elements are within factors of the order of 3, in some other cases the differences are much larger. It shows one of the essential features of the nuclear double-beta decay, namely that case-by-case theoretical studies are needed instead of a global one [17].

5. Conclusions

To conclude, we have presented results on the effective neutrino mass, as obtained from the best-fit mass-mixing matrix determined from the analysis of solar+atmospheric+reactor+ satellite data, and compared them with the values extracted from neutrinoless double-beta-decay experiments.

The effective electron-neutrino mass extracted from the neutrino-related experiments, of the order of 0.24 eV, does not compare with the central value, 0.39 eV, reported in [27] and obtained by using the nuclear matrix elements calculated in [47]. It does not compare, either, with the values given by the standard pnQRPA model, after taking into account the span in the calculated matrix elements.

To explain for the difference between the above results, we have compiled a systematics of the calculated nuclear matrix elements and performed additional pnQRPA calculations. In the case of 76Ge, and if one adopts for the half-life the upper limit suggested in [29], the nuclear matrix elements needed to yield the desired effective neutrino mass are larger than any of the known nuclear matrix elements calculated in the framework of the spherical pnQRPA. This conclusion also holds for the available shell-model results.

The present knowledge of the involved nuclear matrix elements shows that the sensitivity of the experiments is still far from the estimate coming from neutrino oscillations. However, the needed sensitivity is potentially achievable by the next generation of experiments.

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