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UNCERTAINTY ANALYSIS IN STRUCTURAL ENGINEERING

por David I. Blockley

Department of Civil Engineering - University of Bristol, UK

1. INTRODUCTION

We will begin by distinguishing four types of problems. I define a problem as a doubtful or difficult question to which there may be a number of possible answers. Each possible answer is a conjecture which has to be evaluated in a decision procedure.

Type 1 Where all of the consequences of adopting a conjectural solution are known: single decision maker with a single objective i.e. non-chaotic determinism.

Type 2 Where all of the consequences of adopting a conjectural solution have been precisely identified but only the probabilities of occurrence are known: decision theory.

Type 3 Where all of the consequences of adopting a conjectural solution have been approximately identified so that the possibilities of ill defined or fuzzy consequences are known: extension of type 2 with fuzziness of definition.

Type 4 Where only some of the consequences (precise or fuzzy) of adopting a conjectural solution have been identified: real world open problems with incompleteness.

Type 1 problems are the traditional deterministic models of engineering science. The adequacy of these models has always depended on the relationship between the model and reality. Under well defined circumstances, such as the deflection of a simple steel beam on knife edge support with an applied point load, they can give dependable answers. When used to model practical circumstances the results have to be treated with caution so that the answers are on the safe side. However, such calculations have been the basis of the successful application of engineering science for the past 150 years. The recent developments in deterministic chaos have demonstrated that the behaviour of some non-linear deterministic systems can be very complex.

Type 2 problems are the traditional applications of probability theory to closed world problems. In other words the sample space is closed; all conjectures have been identified and the only significant question is which one to choose. in decision theory this can be done by choosing the alternative with maximum expected utility. The important point is that no possibilities outside those identified are considered; it is therefore dependent on the ability of the decision maker to imagine all of them. The final probability figures are then relative measures in comparison with the total set. In a closed world all things are either true or false; there is no room for any other state such as don't know.

Type 3 problems are equivalent to type 2 problems but with ill defined, imprecise or fuzzy state descriptions. The theory of fuzzy sets has allowed a formalization of this state

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description. Thus it is now possible to talk in terms of probabilities of fuzzy events (probability of a man being tall) and of fuzzy probabilities (a high probability). In fact that formalization brings its own problems which I will discuss later.

Type 4 problems are real open world problems where some things may be true, some may be false and some unknown. The central point here is that any measure of the evidence in favour of an alternative must be quite separate from the evidence against that same proposition.

2. Probability Theory

The Kolmogarov axioms of probability theory are well known. For a set $\theta = \{\theta_1, \theta_2, \theta_3 \ldots\ldots\ \theta_n\}$, the probability function P is defined over the power set E so that:

$$P : E \rightarrow [0, 1]$$

must satisfy, for each A, B contained in θ

$$0 <= P(A) <= 1$$

$$P(\phi) = 0, P(\theta) = 1$$

$$P(A \cup B) = P(A) + P(B) \text{ if } P(A \cap B) = 0$$

and so the closed world assumption is

$$P(A) + P(-A) = 1$$

$$\text{Now } 0 <= P(A \cap B) <= \min(P(A), P(B))$$

We define σ such that $P(A \cap B) = \sigma \min(P(A), P(B)$

and σ is the degree of dependence between A and B. Thus

$\sigma = 0$ indicates that A and B are mutually exclusive and

$\sigma = 1$ indicates that $A \subset B$ or $B \subset A$

The parameter σ models the dependency relationship between A and B, knowing σ is equivalent to knowing

$P(A \cap B)$ or $P(A / B)$

Different models have been described (Cui, Blockley 1990) and are

1) Minimum Dependence

$$\sigma = \max(0, P(A) + P(B) - 1) / (\min(P(A), P(B))$$

therefore

$$P(A \cap B) = \max(0, P(A) + P(B) - 1)$$

$$P(A \cup B) = \min(1, P(A) + P(B))$$

2) Independence

$$\sigma = \max(P(A), P(B))$$

$$\text{therefore } P(A \cap B) = P(A).\ P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A), P(B)$$

and this corresponds to the case of maximum entropy.

3) Maximum Dependence

$$\sigma = 1$$

$$P(A \cap B) = \min(P(A), P(B))$$

$$P(A \cup B) = \max(P(A), P(B))$$

These operations correspond with those of fuzzy sets.

3. Fuzziness

The concept of fuzziness is important since it is obvious that in real problem solving there is a need to deal with concepts which cannot or need not be defined precisely in order to solve the problem in question.

The theory of fuzzy sets was extremely important to me in the development of my own thinking since it enabled an escape from the straight jacket of classical probability theory. It helped me to see that it is possible to think about uncertainty differently.

The distinction between fuzziness and probability theory can be drawn in two ways, conceptually and mathematically. The conceptual distinction has been drawn above. The mathematical distinction is that the membership function of fuzzy sets is a mapping from the sample space to the range (0, 1) whereas the probability function as defined above is a mapping from the power set to the range (0, 1).

A fuzzy relation between two sets is a set of points defined on the Cartesian product. The membership values of each of these points may again be interpreted as restrictions on the possible strength of the relationship to allow for imprecision of definition. This is a generalization to a many to many mapping of the normal functional expression y = g (x) which is a one to one or many to one mapping.

4. Interval Probability Theory

An interval number I is a pair of real numbers defining a closed interval [a, b]. In set notation

$$I = \{x \mid a <= x <= b\} = [a, b]$$

An interval variable X is a variable whose value is an interval number. A function of X [a, b] is

$$Y = [\min (f(a), f(b)), \max (f(a), f(b))]$$

In interval probability theory an interval variable is used to represent probability theory, so that

$$P(A) = [S_n(A), S_p(A)]$$

where $S_n(A)$ is the lower bound and $S_p(A)$ is the upper bound of the probability P (A).

The negation is

$$P(\neg A) = [1 - S_p(A), 1 - S_n(A)]$$

If we interpret probability as a measure of belief, then

$S_n(A)$ represents the extent to which it is certainly believed that A is true,

$$1 - S_p(A) = S_n(\neg A)$$

represents the extent to which it is certainly believed that A is false and the value

$$S_p(A) - S_n(A)$$

represents the extent of the uncertainty of belief of whether A is true or false.

The three extremes of [0, 0], [1, 1], [0, 1] therefore represent the cases of 'certainly false', 'certainly true', and 'don't know' or 'unknown'. Thus as described above interval probability theory is an open world model since any constraints on the value of P (A) from evidence in favour of A are quite separate from those constraints from evidence against A.

Cui and Blockley (1990) have provided details of the calculus for interval probability theory. Operations similar to those defined above involving the degree of dependence have been given. In this case the degree of dependence is an interval number. In the case when this dependence is 'unknown' then

$$\sigma = [0, 1]$$

$$P(A \cap B) = [0, (S_p(A) \wedge S_p(B))]$$

$$P(A \cup B) = [S_n(A \cup B), S_p(A \cup B)]$$

where $S_n(A \cup B) = S_n(A) \vee S_n(B)$

$$S_p(A \cup B) = (1 \wedge (S_p(A) + S_p(B)))$$

It was also shown that if one assumes that, in some sense the measures of Baldwin's support logic (Baldwin 1986) and the Dempster - Shafer theory of evidence (Shafer 1976) are equivalent, then the bounds from interval probability theory for union are wider than those of support logic which in turn are wider than those of the Dempster - Shafer theory.

5. Fuzziness and Probability

Engineers are familiar with the idea that physical systems can be modelled in various ways. Part of an engineers technical capability has to be able to choose appropriate theoretical models. It is perhaps surprising therefore that it has been widely held that uncertainties and degrees of belief about those uncertainties can only be modelled in one way i.e. using Bayesian probability theory. Probability theory and the theory of fuzzy sets are two special cases of the more general results given earlier.

Fuzzy set theory was, of course, designed to handle cases where sets do not have sharp boundaries. There seem to be three problems with the theory of fuzzy sets.

Firstly, the philosophical problem is that in order to define a fuzzy set one needs to use numbers to define the membership values, or the indicator functions, which themselves have to be defined in terms of set theory. Thus the definition of a crisp set is required for the definition of number and the distinction between the sample space and the power set is all that is left.

The second problem is concerned entirely with that distinction. Fuzzy sets are defined in terms of measures which themselves have be defined. For example the concept of a tall man has to be defined in terms of numbers of metres. The definition of a sample space depends on the nature of the problem to be solved. We can adopt a systems approach and think of all concepts as 'holons' in a hierarchical system of various levels of precision of definition. If we look upwards towards the infinite vague unity of the universe then any concept is a part, and looking downwards to the precise infinitesimals of the universe any concept is a whole. The sample space represents a choice of holons which are appropriate for the problem at hand. In problems where the sample space consists, for all practical purposes of mutually exclusive, non-interactive concepts, then classical probability theory is sufficient. However, when it is difficult to choose a sample space with these characteristics then estimates of inter-dependence have to be obtained. In a sense fuzzy set theory is an attempt to change the level of definition (in fact to artificially induce vagueness) in order to enable more efficient problem solving. There seems therefore to be little point in arbitrarily choosing to define the meaning of a fuzzy definition in terms of lower level concepts using another functional mapping; why use two functions instead of one?

The third problem is the nature of the calculus of the theories. As shown earlier a general treatment of probability theory shows that the max, min calculus of fuzzy set theory is at one end of a range of possible results.

For the purpose of practical computation, the interval probability theory as a measure on concepts arranged in a hierarchical structured knowledge base, is a simple open world of concepts which are more or less precisely defined. At high levels in a knowledge base, concepts will be vague and will therefore tend to attract high levels of evidential support, but at the expense of information content. At low levels the concepts will be precise and therefore of high information content, but with consequently lower levels of support.

6. Conclusions

1. Four types of problems have been identified, deterministic, probabilistic, probabilistic with fuzzy concepts and incomplete. The first three are closed world models and the fourth is an open world model.

2. A closed world model is one where every possibility has been identified. An open world model is one where it is recognized that there may be some possibilities which have not been identified.

3. A measure of degree of dependence bounded on the interval (0, 1) has been defined.

4. A theory of Interval Probability has been introduced and suggested as a relatively simple and robust theory for practical computation.

5. Three problems with the theory of fuzzy sets have been discussed. They are concerned with the philosophical basis, the choice of sample space and the choice of calculus.

7. REFERENCES

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